The Effects of Destructive Interference and Wasted Transmissions on the Uniform-Traffic Capacity of Non-Bus-Oriented Single-Hop Interconnections

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***Abstract*—The uniform-traffic capacity of switchless, non-bus-oriented, fiber-optic single-hop interconnections among N stations, each equipped with a small number of transmitters and receivers, can be as high as $\Theta(\log_2 N)^*$ concurrent transmissions on a single wavelength with round-robin scheduling in a time-slotted system. However, their capacity with the slotted ALOHA access scheme does not increase with N. (The capacity of bus-oriented interconnections, in contrast, varies across time-slotted access schemes by, at most, a factor of e.) This paper quantifies the contribution of several factors to capacity. Merely avoiding destructive interference with ongoing receptions contributes, at most, a factor of e over slotted ALOHA, the same as in bus-oriented interconnections. For an interconnection among two-transmitter, single-receiver stations, whose capacity is $\log_2 N$ with global scheduling and $2/e$ with slotted ALOHA, also avoiding transmissions to blocked receivers increases capacity to, at most, $\log_2 \log_2 N$. These results suggest that the added complexity of non-bus-oriented SHI's may be warranted only if they are operated in ways that permit the selection of "good" combinations of (source, destination) pairs for concurrent transmission, and further research should focus on those.**

***Index Terms*— Fiber optic networks, single-hop interconnections, shared directional multichannel, capacity, multiple access, local area networks.**

I. Introduction

A. Background

SINGLE-HOP interconnections (SHI's) are static, switchless interconnections that provide a (possibly shared) path between any two stations at all times. The most prominent SHI topology is the single broadcast channel, or bus, which is used in local area networks (LAN's) such as Ethernet. The advantages of SHI's include simplicity, as well as the capability to permit each (source, destination) pair to communicate at a different rate, depending only on their equipment and not imposing extra cost on the interconnection fabric or on other stations. This is particularly attractive in the fiber-optic domain.

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The single bus, albeit simple and flexible, permits at most one ongoing transmission. To permit concurrent noninterfering transmissions, one can give up single-hop connectivity or use switches to dynamically route concurrent messages to their respective destinations without conflict. These approaches have been employed in the telephone network as well as in wide-area networks. More recently, they are being applied to LAN's in the form of switching hubs. These approaches can dramatically increase performance, but can be costly. In the last several years, various researchers have explored ways of permitting concurrent transmissions while retaining static, single-hop connectivity through a passive fabric [1]–[10].

One way of permitting concurrent transmissions while retaining static, single-hop connectivity is to employ wavelength division multiplexing using tunable transmitters and/or receivers [8], [9], [11]. Alternatively, one can equip each station with multiple transmitters and/or receivers, and construct interconnections that achieve spatial separation among concurrent transmissions, or separation by wavelength without requiring tunable components [12]. One way of combining spatial- and wavelength-separation among message paths is to use "λ-routing" [13], i.e., employ passive multiport components that route a message based on its wavelength. The discussion in this paper is cast in terms of a single wavelength and spatial separation, which permits the use of the simplest, least expensive fiber-optic components.

B. Terminology

Consider an SHI connecting N_S source stations (SS's), each equipped with c_T transmitters, to N_D destination stations (DS's), each equipped with c_R receivers. We use $(c_T, c_R; N_S, N_D)$ to denote the size of such an SHI. Whenever $N_S = N_D = N$, one can think of N bidirectional stations. The interconnections considered in this paper will all be "equal-degree," "single-path" SHI's: every transmitter is heard by the same number of receivers and every receiver can hear the same number of transmitters, and there is a single path between any two stations.

We distinguish between merely *hearing* a transmission and actually *receiving* a packet: a receiver is engaged in the reception of a packet iff it hears the transmission of this packet, hears no other transmissions, and is the packet's destination.

We assume a slotted time system and single-slot packets. For random access schemes, a single time slot is considered. We define the uniform-traffic capacity of a given SHI when

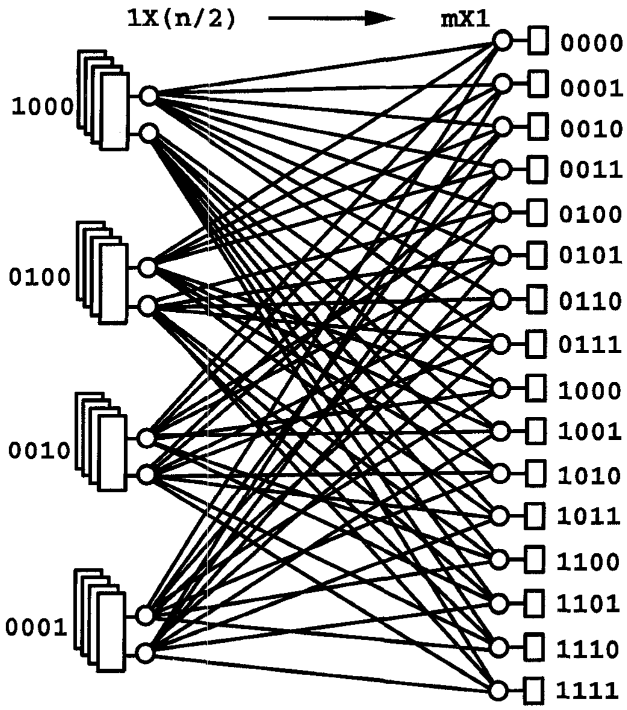

Fig. 1. An interconnection of 2^k two-transmitter SS's and 2^k single-receiver DS's ($k = 4$). The SS's are of k "types," each wired differently, with $2^k/k$ identically-wired SS's in each type. Rectangles and circles denote stations and star couplers, respectively [5].

operated with a given access scheme to be the maximum (over offered load) mean number of concurrent noninterfering transmissions attainable with this access scheme, under a constraint of an equal (mean) amount of traffic between any pair of stations. Uniform-traffic capacity is the only performance measure considered in this paper, it being a natural first step toward understanding the behavior of general SHI's.

C. Topologies of Single-Hop Interconnections

SHI's can be classified as *bus-oriented* or *non-bus-oriented.* Bus-oriented SHI's can be described as a collection of "buses": Any transmitter or receiver is connected to exactly one bus, and for every (SS, DS) there is a bus to which they are both connected. Such SHI's with up to $c_T \cdot c_R$ equally-populated buses can be constructed [3], [4]. The capacity of a bus-oriented SHI with equally-utilized buses under any given access scheme equals the number of buses times the capacity of a single bus. When all buses are equally loaded, as is the case for these SHI's with a uniform traffic pattern, the capacity with a "perfect" access scheme is, thus, $c_T \cdot c_R$. In [7] and [14], it was formally proved that $c_T \cdot c_R$ is an upper bound on the uniform-traffic capacity of any bus-oriented SHI even if symmetry constraints on the construction are removed. Using the capacity with slotted ALOHA as a practical lower bound, capacity thus varies over the range of access schemes by a factor of $e \approx 2.7$ and is independent of the number of stations. (If $c_T = c_R = c$ and a station must connect its transmitters and receivers to the same buses, the maximum number of buses is slightly lower: $c^2 - c + 1$. The number of stations, N, is assumed to be much larger than c^2 [12].)

Non-bus-oriented SHI's are based on a *shared directional multichannel* [5]: a set of inputs and a set of outputs, to which one connects transmitters and receivers, respectively, and a specification of connectivity between inputs and outputs. When a transmitter transmits, its signal reaches all receivers connected to it. Unlike with buses, the sets of receivers reachable from any two transmitters do not have to (but may) be identical or disjoint. Throughout much of the paper, we will refer to $(2, 1; 2^k, 2^k)$ SHI's whose uniform-traffic capacity with a fixed round-robin schedule is $k = \log_2 N$ [5]. Fig. 1 depicts a bipartite graph representation of such an SHI for $k = 4$.

The construction of non-bus-oriented SHI's requires directional couplers, available for various media. Fiber-optics is especially attractive, since the commonly used transmissive star couplers only propagate signals in their original direction and can thus be employed to construct a shared directional multichannel. (In contrast, a binding post to which copper wires are soldered would not work.) Shared directional multichannels can be encountered in wireless networks as well, due to imperfections in the desired connectivity (e.g., blocking by a mountain [15] or the result of limited transmission range [16]) rather than a design with any special features. Designs such as the one depicted in Fig. 1 in wireless technology would require antennas with $N/2$ narrow beams, and are thus impractical. The use of holographic techniques may, in the future, permit a wireless (optical) design of a "wiring closet."

The additional (non-bus-oriented) topological degree of freedom has permitted the construction of SHI's among stations, each of which is equipped with a small fixed number of transmitters and receivers, whose uniform-traffic capacity with a round-robin schedule increases polylogarithmically with the number of stations. For example, if 160 stations are each equipped with two transmitters and two receivers, an SHI with a uniform-traffic capacity of 25 can be constructed [5]. This is a dramatic improvement over the highest capacity attainable with bus-oriented SHI's, four in this example. Moreover, the high-capacity SHI's can be laid out using a number of fiber segments that is only slightly larger than for bus-oriented ones, and with a path loss of only N (versus N/c for bus-oriented SHI's) [17]–[19]. The small difference in path loss is of particular importance due to its effect on the permissible transmission rate. It has also been shown that concurrency N can be attained with $N^{1/3}$ transmitters and receivers per station [8]. This was achieved by extending a construction in [5] and [7] for $N = 8$, and then repeatedly applying a composition method described in [5]. A similar result can be obtained by composing two small designs that were described in [5] and [7]. In the remainder of this paper, we focus on SHI's among stations with a small number of transmitters and receivers.

Unfortunately, the high capacity of non-bus-oriented SHI's was derived only for a fixed round-robin transmission schedule. Worse yet, each time slot in the schedule is allocated to specific (source, destination) pairs. This is in contrast with schedules for bus-oriented SHI's, wherein, permission to transmit is granted to specific sources with no restriction on the destination. With N stations, the length of the schedule round is, therefore, N^2 divided by the degree of concurrency. This causes undesirably high low-load delay (half of a very long round on average), and extreme sensitivity of capacity to the traffic pattern.

Random access schemes such as slotted ALOHA [20] facilitate flexible allocation of the transmission media and sharply reduce low-load delay. However, the uniform-traffic capacity of any single-path, equal-degree SHI is $\frac{1}{e} \cdot c_T \cdot c_R$ with Slotted ALOHA. This is much smaller than the capacity with round-robin schedules, which increases with N as $(\log_2 N)^{c_T+c_R-2}$ [5]. Schemes such as CSMA and CSMA-CD [21] are meaningless for non-bus-oriented SHI's.

The difference between the dynamic range of capacity of non-bus-oriented SHI's across the range of access schemes and that of bus-oriented ones is, thus, dramatic: a factor of $\Theta((\log_2 N)^{c_T+c_R-2})$ versus a factor of $e \approx 2.7$. The dynamic range of delay at very low loads is also much larger for non-bus-oriented SHI's.

The strive for higher performance of non-bus-oriented SHI's can proceed in two directions: 1) searching for access schemes that retain the high capacity of round-robin scheduling while achieving lower delay at light loads, and 2) searching for access schemes that retain the low delay at light loads while achieving higher capacity than slotted ALOHA. This paper sheds light on the prospects of the second approach.

D. Contributors to the Uniform-Traffic Capacity of Non-Bus-Oriented SHI's

For a transmitted packet to be received successfully, the intended receiver must not hear any other transmission throughout the reception. Consequently, there are three contributors to achieving a large number of concurrent, successful receptions:

1) Avoiding destructive interference of new transmissions with ongoing receptions.
2) Avoiding transmissions addressed to blocked receivers. (We refer to these as *wasted transmissions* and note that they may also block additional receivers.)
3) Picking large subsets of mutually-noninterfering (SS, DS) pairs for concurrent transmission.

The first contributor is meaningful in all SHI's. The two others, however, are unique to non-bus-oriented SHI's, since in bus-oriented ones the second is implied by the first and the third is essentially guaranteed under heavy load with a efficiently large number of stations.

Access schemes can be characterized by the extent to which they feature the different contributors. Slotted ALOHA, for example, features none of them, whereas, optimal round-robin schedules feature all of them. Busy-tone multiple-access [15], whereby prospective transmitters are aware of ongoing transmissions with whose reception they would interfere, features the first contributor to an extent that depends on propagation delay and race conditions. However, it does not feature the other two.

E. Overview of This Paper

The purpose of this paper is to quantify the contributions of the aforementioned factors. Since different access schemes can be characterized by the extent to which they feature the aforementioned contributors, the findings of this paper can assist in deciding whether the additional complexity of non-bus-oriented SHI's is warranted if operated with a given access scheme.

It is not the goal of this paper to suggest actual access schemes. However, the incorporation of the first two contributors will be expressed "algorithmically" as access schemes, wherein, transmitters execute the access protocol one after the other in some random order at the beginning of every time slot. This approach provides an intuitive, unambiguous definition and lends itself to analysis.

Since schemes with growing sophistication tend to feature the contributors in the order in which they were presented, we will quantify the contribution of the first one in isolation, and then quantify the impact of adding the second one. (The uniform-traffic capacity with all three is known [5].) Throughout this paper, we assume a slotted system with single-slot packets and independent operation from slot to slot. This is necessary in order to prevent the third contributor from creeping in through the combination of incremental changes in the set of ongoing transmissions and the remaining two contributors [22].

The remainder of this paper is organized as follows. The effect of destructive interference is studied in Section II for a broad range of SHI's. In Section III, the discussion is restricted to the $(2, 1; 2^k, 2^k)$ SHI's, for which an upper bound on the uniform-traffic capacity attainable by also avoiding wasted transmissions is derived, and Section IV offers concluding remarks.

II. Capacity with No Destructive Interference

The derivations will be made for a more general setting of a shared directional multichannel, and then applied to SHI's among stations with multiple transmitters and receivers.

Transmission Model: At the beginning of each time slot:

- All backlogged transmitters are ordered at random.
- Each transmitter, in its turn, checks the state of all the receivers that can hear it.
  - —If any such receiver is engaged in the reception of a packet (not merely hears one), the transmitter is considered *blocked* and refrains from transmitting;
  - —Otherwise, it picks a random destination from among those that can hear it and transmits a packet intended for it (the choices are assumed to be independent from transmitter to transmitter and from slot to slot).

Note that a transmitter may unknowingly transmit a message addressed to a *blocked receiver.*

The foregoing algorithmic expression of the avoidance of destructive interference with ongoing receptions is equivalent to the well-known busy-tone multiple-access (BTMA) scheme [15] in an idealized, slotted setting.

Since the reception of an "early" transmission cannot be interfered with (at its intended receiver) by a later one, throughput is monotonically nondecreasing with offered load. To derive the uniform-traffic capacity, we will, therefore, assume that all transmitters wish to transmit in every slot and have backlogged messages for all their prospective destinations (*heavy load*).

Theorem 1: The uniform-traffic capacity of any single-path, equal-degree SHI without destructive interference and with destinations picked randomly by transmitters, independently from transmitter to transmitter and from slot to slot, is at most $c_T \cdot c_R$.

Proof: Consider a shared directional multichannel with t inputs and r outputs, such that each input is connected to exactly d_T outputs and each output is connected to exactly d_R inputs. Clearly, $t \cdot d_T \equiv r \cdot d_R$.

A transmitter transmits only if none of the receivers that can hear it is engaged in a reception. When it decides to transmit, it does not know whether any of these receivers is hearing a transmission (and is, thus, blocked). Consequently, it picks the destination with equal probability from among the d_T receivers that can hear it. The probability that the first transmission heard by any given receiver is intended for it is, therefore, $1/d_T$. Making the optimistic assumption that every receiver hears a single transmission, capacity is thus bounded from above by r/d_T. Casting this result in the context of the SHI's among stations, $r = N_D \cdot c_R$ and $d_T = N_D/c_T$. Consequently, $S \leq c_T \cdot c_R$. □

It is clear from Theorem 1 that merely avoiding destructive interference with ongoing receptions leaves capacity constant in N and exactly equal to that of the highest-capacity bus-oriented SHI's, so the capacity advantage of non-bus-oriented SHI's over bus-oriented ones must be due entirely to the remaining contributors. This will be the topic of the next section, in which the discussion is limited to a particular type of $(2,1;N,N)$ SHI's.

III. Capacity with No Unsuccessful Transmissions

In this section, we assume that wasted transmissions are also avoided, so only successful transmissions take place. Under this assumption, we derive the uniform-traffic capacity of $(2,1;N,N)$ SHI's. We begin by describing the construction of the $(2,1;2^k,2^k)$ SHI's whose uniform-traffic capacity with global scheduling is k, as depicted in Fig. 1, and their properties. Analysis will first be carried out for a $(2,1;k,2^k)$ SHI and will then be extended to $(2,1;2^k,2^k)$.

A. $(2,1;2^k,2^k)$ *SHI's [5]*

Consider initially an SHI interconnecting k SS's, each with two transmitters, and 2^k DS's, each with a single receiver. Let each SS be labeled with a k-bit vector containing a single "one," and each DS—with a k-bit vector. Let W be the wiring matrix, each of whose rows and columns correspond to an SS and to a DS, respectively. $W(s,d) \in \{0,1\}$ specifies which of the two transmitters of s can be heard by d's receiver. The receiver needn't be specified, since there is a single receiver per station. A wiring matrix that attains a uniform-traffic capacity of k when operated with an appropriate round-robin transmission schedule is $W(s,d) = (s \cdot d)$. (Inner product of the two k-bit vectors, modulo 2.) Thus, each SS partitions the DS's based on a different bit in their k-bit label [5]. Fig. 1 depicts such an SHI for $k = 4$. (Refer only to the top "layer" of SS's.)

To obtain a $(2,1;2^k,2^k)$ SHI with the same concurrency, $2^k/k$ groups of k SS's each are created. (See Fig. 1.) The ith station in every group, $i = 0,1,\cdots,k-1$, is wired identically to the equally-numbered SS in the $(2,1;k,2^k)$ SHI. We will refer to SS's with identical wiring as being of the same *type*. The use of dummy stations to construct SHI's whose size is not a power of two was discussed in [17].

In this paper, we use an expanded wiring matrix, W', whose rows and columns correspond to individual transmitters and receivers, respectively. $W'(i,j) = 1$ iff the corresponding transmitter and receiver are connected, and zero otherwise. Such a matrix of size $(2k \times 2^k)$ is used to describe the $(2,1;k,2^k)$ SHI. In the remainder of this paper, "wiring matrix" refers to the expanded matrix unless stated otherwise. The matrix for $k = 4$ is shown in Example 1.

B. *Capacity of the* $(2,1,k,2^k)$ *SHI's*

We next derive the uniform-traffic capacity when there are neither destructive interference nor wasted transmissions.

Transmission Model: At the beginning of each time slot:

- All backlogged transmitters are ordered at random.
- Each transmitter, in its turn, checks the state of all the receivers that can hear it.
  - —If any such receiver is engaged in the reception of a packet (not merely hears one), the transmitter is considered *blocked* and refrains from transmitting;
  - —Otherwise, it lists those receivers that are not hearing any transmissions, picks one of them randomly, and transmits a packet intended for it. (The choices are assumed to be independent from transmitter to transmitter and from slot to slot.)

Here, all transmissions are received successfully. Throughput is again monotonically nondecreasing with offered load, so capacity is derived under the assumption that every transmitter always has backlogged packets for every receiver that can hear it.

In the $(2,1;k,2^k)$ SHI, the two transmitters of any given SS jointly reach all receivers. Allowing an SS to transmit with both of its transmitters would thus block all receivers and limit the throughput to two. Therefore, we initially assume that a transmitting SS disables its remaining transmitter. This assumption has a minor effect on capacity, as will become apparent once it is relaxed as the derivation is extended to $(2,1;2^k,2^k)$.

Considering again a single time slot, the following lemmas help establish important properties of the submatrix of W' that corresponds to the remaining free transmitters and receivers following the beginning of a new transmission. Lemma 2 describes the impact of the first transmission in a time slot. The impact of subsequent transmissions (in the same time slot) is considered in Lemma 3. The process stops when all transmitters are blocked (their transmission would result in destructive interference) or disabled. Detailed proofs appear in [22], and the intuition for them can be obtained from Example 1.

Lemma 2: The $(k-1) \times (2^{k+1})$ wiring matrix corresponding to the portion of the $(2,1,k,2^k)$ SHI that remains free after the first transmission consists of 2^{k-1} different columns that jointly comprise all possible $(k-1)$-bit vectors.

Lemma 3: The wiring matrix corresponding to the transmitters and receivers that remain free after any transmission comprises all possible column bit-vectors of the appropriate size, each of which appears an equal number of times.

Corollary 4: At least one free receiver is connected to every remaining free transmitter in the $(2,1;k,2^k)$ SHI when a station may transmit with, at most, one of its transmitters in any given time slot.

It is important to realize that while the number of free receivers is halved by each transmission regardless of its source and destination, the number of additional blocked transmitters depends on the identities of the members of the free subnetwork and on the receiver of the transmitted packet. Also, in view of Lemma 3 and the fact that the multiplicity of identical columns in the wiring matrix does not affect throughput, the size of a subnetwork of a $(2,1;k,2^k)$ SHI will be stated as l, the number of free transmitters. $S_{(l)}$ will be used to denote the uniform-traffic capacity of such a subnetwork.

Lemma 5: The probability mass function (pmf) of the number of free transmitters remaining after a transmission in a subnetwork of size l is binomial $[0, l-1]$ with mean $\frac{l-1}{2}$.

Proof: Given the wiring matrix corresponding to the subnetwork of remaining free transmitters and receivers, the number of transmitters that become blocked following the next transmission depends only on the identity of its receiver. From this and Lemma 3, it follows that, given l free transmitters, the probability that the engagement of the destination of the next transmission in reception will block $(l-1-m)$ transmitters $(m = 0, 1, \cdots, l-1)$ equals the probability that a randomly chosen $(l-1)$-bit vector contains exactly m zeros. This probability is $2^{-(l-1)}\binom{l-1}{m}$. □

Corollary 6: The probability that a transmission in a subnetwork of size l will leave a subnetwork with m free transmitters is symmetric about $\frac{l-1}{2}$, and decreases as one moves away from its mean in either direction.

Theorem 7: The uniform-traffic capacity of the $(2,1,k,2^k)$ SHI without destructive interference or wasted transmissions is

$$S = 1 + S_{(k-1)}$$

where

$$S_{(l)} = 1 + 2^{-(l-1)} \sum_{m=0}^{l-1} \binom{l-1}{m} S_{(m)}, \quad l = 1, \cdots, k-1; \quad S_{(0)} = 0.$$

Proof: The "one" stands for the first transmission. Having established in Corollary 4 that the number of concurrent transmissions in any given slot is limited by transmitter availability, it suffices to determine the expected number of transmissions required to block all transmitters. Having eliminated the transmitting transmitter, $l-1$ transmitters may be free. By Lemma 5, the probability that m of these $l-1$ transmitters remain free is $2^{-(l-1)} \cdot \binom{l-1}{m}$. Hence the expression for $S_{(l)}$. □

Corollary 8: Capacity does not depend on the identity of the transmitting transmitter in each step; it only depends on the chosen destination of its transmission.

We next use a detailed example to illustrate the operations during a time slot and to provide intuition for the lemmas and their proofs.

Example 1: Consider the expanded wiring matrix, W', for the $(2,1;4,16)$ SHI that was depicted in Fig. 1. See matrix at the bottom of this page.

By Lemma 2, we may assume without loss of generality that the first transmission is from the first transmitter (row) to the first receiver (column). The remaining matrix corresponds to a subnetwork with $l = k-1 = 3$ free transmitters of different SS's, as well as 8 free receivers

$$W'(\text{subnet}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & 0 & 0 & 1 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & 0 & 1 & 0 & 1 \end{pmatrix} \subset W'.$$

The uniform-traffic capacity of the original SHI is thus

$$S = 1 + S_{(3)}.$$

By Corollary 8, we may again assume, without loss of generality, that the second transmission is by the first transmitter of the remaining network. This transmission will block the two remaining transmitters if the destination is seven (probability $\frac{1}{4}$), one free transmitter will be left if the destination is five or six (probability $\frac{2}{4}$), and two—if the destination is four (probability $\frac{1}{4}$). Capacity is therefore

$$S = 1 + 1 + \frac{1}{4} \cdot S_{(0)} + \frac{2}{4} \cdot S_{(1)} + \frac{1}{4} \cdot S_{(2)}.$$

It is obvious that $S_{(0)} = 0$ and $S_{(1)} = 1$. If two free transmitters are left, the remaining matrix is

$$W'(\text{subnet}) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{pmatrix} \subset W'.$$

Again, without loss of generality, we assume that the third transmission is by the first transmitter of the remaining

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1}
\end{pmatrix}$$

network. This transmission will block the last transmitter with probability $\frac{1}{2}$. Consequently

$$S_{(2)} = 1 + \frac{1}{2} \cdot S_{(0)} + \frac{1}{2} \cdot S_{(1)} = 1.5$$

and the capacity of the $(1, 2; 4, 16)$ SHI without wasted transmissions is, thus, $S = 2.875$.

C. Explicit Bounds for $(2, 1; k, 2^k)$ SHI's

The recursive expression for capacity, while accurate, offers limited insight. Therefore, we next derive explicit expressions for an upper and a lower-bound on capacity.

Lemma 9: Bounds on capacity can be attained by replacing the Binomial pmf of the number of remaining free transmitters after a transmission in a subnetwork of size l, as follows: 1) assuming that the remaining subnetwork comprises exactly $\frac{l-1}{2}$ free transmitters (equal to the mean) yields an upper bound; 2) assuming a uniform$[0, l-1]$ distribution yields a lower bound.

Proof: Follows from Corollary 6, the concavity of the capacity function and Jensen's inequality. □

Lemma 10: The capacity of the $(2, 1; k, 2^k)$ SHI with no destructive interference or wasted transmissions and with each station transmitting with, at most, one of its transmitters is bounded as follows:

$$1 + \ln k < S \leq 1 + \lceil \log_2 k \rceil, \quad k \geq 2.$$

Proof: The "one" stands for the first transmission, which leaves $k-1$ free transmitters, all belonging to different SS's.

1) Upper Bound: Based on Lemma 9 and ignoring integer constraints, we take the subnetwork remaining after a transmission in a subnetwork of size l to be of size $\frac{l-1}{2}$, thus

$$S_{(l)} \leq 1 + S_{\left(\frac{l-1}{2}\right)}.$$

Applying this $\log_2 k$ times and recalling that $S_{(1)} = 1$ and $S_{(0)} = 0$ yields

$$S \leq 1 + \log_2 k, \quad k > 2.$$

Since S is monotonically nondecreasing in k, the integer constraints can be accommodated by deriving a slightly looser bound as follows: k is replaced with the next larger "nice" value of k, namely, one for which successive applications of subtraction of one and division by two produce a sequence of integers. Since the upper bounds on S for successive "nice" values of k differ by exactly one, the looser bound on S is simply the ceiling of the noninteger one.

2) Lower Bound: Here too, based on Lemma 9, we take the size of the remaining subnetwork after a transmission in a subnetwork of size l to be distributed uniformly in the range $[0, l-1]$. See [22] for a detailed proof.

Finally, it can be shown by direct computation that $S = 2$ for $k = 2$, which completes the proof.

D. Extension to $(2, 1; 2^k, 2^k)$ Interconnections

Since the wiring of all SS's of any given type is identical, so is the blocking situation for them. However, even if the SS that makes the first transmission disables its other transmitter, the remaining SS's of the same type do not. In order to extend the capacity bounds to this case, we next relax the assumption that the first SS disables its other transmitter.

Lemma 11: The unused transmitter of the SS that makes the first transmission and the unblocked transmitter of every SS of its type all become blocked following the second transmission, regardless of its source and destination.

Proof: The second transmission is intended for an unblocked receiver. This receiver did not hear the transmission of the first transmitter. Since the two transmitters of every SS jointly reach all receivers, however, the receiver of the second transmission must be connected to the other transmitter of the SS of the first transmission and to the unblocked transmitters of every SS of its type. Therefore, they all become blocked.□

Theorem 12: The capacity of the $(2, 1; 2^k, 2^k)$ SHI with no destructive interference or wasted transmissions is bounded as follows:

$$\frac{1}{k} \cdot 2 + \frac{k-1}{k} \cdot (1 + \ln k) < S \leq 1 + \log_2 k, \quad k \geq 2.$$

Proof: From Lemma 11, it follows that a single SS, or two SS's (of the same type), can jointly block all receivers only during the second transmission. The probability of this event is $\frac{(2^k/k)-1}{2^k-1} \approx 1/k$. Combining this with Lemma 10 and assuming $k \geq 2$ produces the result. □

Remark: The error introduced by the approximation of $1/k$ slightly relaxes the bound but retains correctness. Also, it can readily be shown by direct computation that $S = 2$ for $k = 1, 2$.

This result is theoretically quite intriguing, suggesting that the uniform-traffic capacity with an access scheme that operates independently from slot to slot can increase with the number of stations. From a practical perspective, however, the increase is very slow $(\log \log N)$ and attaining it requires a very complicated access scheme in an ideal setting.

IV. Conclusion

This paper used the equivalent of "idealized" versions of access schemes to quantify the contribution of various properties of access schemes to the uniform-traffic capacity of certain non-bus-oriented SHI's.

The mere avoidance of interference with ongoing successful receptions was shown to have the exact same effect as in bus-oriented SHI's, thus, offering no capacity advantage to non-bus-oriented SHI's over the highest-capacity bus-oriented ones of the same size. For the particular SHI's among two-transmitter, single-receiver stations $(2, 1; N, N)$, also avoiding transmission to blocked receivers raised capacity to, at most, $\log_2 \log_2 N$. Attaining higher capacity, up to $\log_2 N$ in this case, thus requires some way of choosing large sets of mutually noninterfering (source, destination) pairs.

The capacity results with no destructive interference apply to any single-path, equal-degree SHI, but the analysis when also avoiding wasted transmissions applies only to the $(2, 1; N, N)$ SHI's. The determination of the uniform-traffic capacity of arbitrary SHI's with no wasted transmissions thus remains an open problem. Nevertheless, this paper gives important insight into the contribution of several defining features of access schemes to the capacity of non-bus-oriented SHI's. Also, practicality of the access scheme not withstanding, an increase in capacity of an SHI with an increase in network size without cross slot scheduling is an intriguing theoretical result.

The findings of this paper suggest (and prove for a specific case) that the capacity-advantage of non-bus-oriented SHI's over bus-oriented ones cannot be attained without the use of schemes that feature the ability to pick "good" sets of (source–destination) pairs. Therefore, recommended directions for further research include ways of shortening the schedule round in round-robin schemes, reservation-based schemes, and perhaps unslotted (continuous-time) schemes in which network state evolves incrementally over time.

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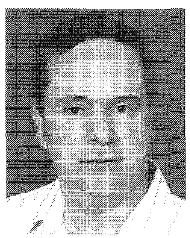

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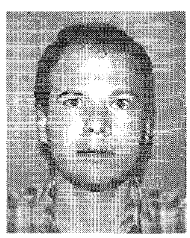

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