# Operation of Non-Bus-Oriented Single-Hop Interconnections: Busy-Tone Multiple Access and Folded Schedules * 

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#### Abstract

The use of transmissive star couplers permits the construction of switchless, non-bus-oriented single-hop interconnections. By equipping each station with a small, fixed number of transmitters and receivers, such interconnections whose uniform-traffic capacity increases with the number of stations can be constructed and can be laid out efficiently. However, the fixed round-robin schedules used to attain the high capacity result in prohibitive delays at low loads, and throughput is very sensitive to skews in traffic load. In this paper, two techniques for mitigating these problems are presented along with an initial assessment of their impact and limitations.


Key words and phrases: fiber optic LANs, shared directional multichannel, multiple access, folded schedule, busy-tone multiple-access, local area networks.

## 1. Introduction

Single-hop interconnections, SHI's for short, are static, switchless interconnections which provide a (possibly shared) path among any pair of stations at all times. The most prominent such interconnection is the bus, which is used for LANs such as Ethernet. While flexible and simple, the single bus permits at most one ongoing transmission and is thus a bottleneck.

One way of solving this problem is giving up the single-hop connectivity or adding switches to dynamically partition the network and route concurrent messages to their respective destinations without conflict. This approach has been employed in the telephone network as well as in wide-area networks. More recently, it is also being applied to LANs in the form of switching hubs. However, this comes at a cost.

In the last several years, various researchers have explored ways of permitting concurrent transmissions while retaining static, single-hop connectivity through a passive fabric

[^0][1][2][3][4][5][6] [7][8][9][10][11]. Advantages include simplicity, as well as the ability to permit each (source, destination) pair to communicate at a different rate, depending on their equipment. This is particularly attractive in the fiber-optic domain. To permit concurrent transmissions while retaining static, single-hop connectivity, one must either use tunable components or equip each station with multiple transmitters and/or receivers. The discussion in this paper is limited to non-tunable components.

Initial results involved the use of "bus-oriented" interconnections. These can be described as a collection of "buses", such that any given transmitter or receiver is connected to a single bus, and for any two stations there is at least one bus to which they are both connected. If each station is equipped with $c_{T}$ transmitters and $c_{R}$ receivers, it was shown that up to $c_{T} \cdot c_{R}$ equally populated buses can be constructed, such that any two stations have a bus in common [3][4]. If $c_{T}=c_{R}=c$ and a station must connect its transmitters and receivers to the same buses, the maximum number of buses is slightly lower: $c^{2}-c+1$. The number of stations, $N$, is assumed to be greater than $c^{2}$ [12].

The use of transmissive star couplers permits the construction of "non-bus-oriented" SHI's. With this shared directional multichannel [6], unlike with buses, the sets of receivers reachable from any two transmitters needn't be identical or disjoint. This additional degree of freedom has permitted the construction of SHI's whose uniform-traffic capacity increases polylogarithmically with the number of stations. For example, if 160 stations are each equipped with two transmitters and two receivers, an SHI that permits 25 concurrent transmissions can be constructed [6]. This is a dramatic improvement over the highest capacity attainable with bus-oriented interconnections, 4 in this example. Moreover, the high-concurrency SHI's can be laid out with only a fixed number of fiber segments per station, and with a path loss of only $N$ (the number of stations) [13][14][15]. While our focus is on the case of a small, fixed number of transmitters and receivers per station, it has recently been shown in [9] that concurrency $N$ can be attained with $N^{1 / 3}$ transmitters and receivers per station. This was achieved by extending a construction in [6] and [8] for $N=8$, and then repeatedly applying a composition described in [6]. A similar result can be obtained by simply composing two small designs that were described in [6] and [8].

Unfortunately, the high degree of concurrency of SHI's was demonstrated only using a fixed round-robin transmission schedule. Worse yet, each schedule slot is allocated to specific (source, destination) pairs. This is in contrast with conventional TDMA, wherein the right to transmit is given to specific sources with no restriction on the destination. With $N$ stations, the length of the schedule round is therefore $N^{2}$ divided by the degree of concurrency. This causes undesirably high low-load delay (half of a very long round on average), and extreme sensitivity of throughput to traffic pattern. For example, the schedule round for an interconnection among $N$ stations, each with two transmitters and a single receiver, would consist of $N^{2} / \log _{2} N$ time slots: in each time slot, $\log _{2} N$ specific (source,destination) pairs would be allowed to communicate.

In this paper, we begin to address these problems by exploring other ways of operating the interconnections. In section 2, we introduce our notation and briefly describe an SHI with capacity $\log _{2} N$. In section 3, we consider "random access" techniques and derive some bounds. In section 4, we introduce the technique of "schedule folding", and show the extent to which it can be applied. Section 5 offers conclusions and directions for future research.

## 2. Specification of an SHI: wiring and schedule

Consider a set $S=\left\{s_{1}, \ldots, s_{N_{S}}\right\}$ of $N_{S}$ source stations (SS's) communicating with a set $D=\left\{d_{1}, \ldots, d_{N_{D}}\right\}$ of $N_{D}$ destination stations (DS's) in the following way: each station in $S$ has $c_{T}$ transmitters which we index by a set $T,|T|=c_{T}$, and each station in $D$ has $c_{R}$ receivers, indexed by $R,|R|=c_{R}$.
We will restrict the discussion here to SHI's that provide a single path between each SS and every DS. Thus, for each pair of stations $s \in S$ and $d \in D$, exactly one transmitter of $s$ is connected to exactly one receiver of $d$. Denote the index of that transmitter by $W_{1}(s, d) \in T$, and the index of that receiver by $W_{2}(s, d) \in R$. Let $W$ be the $N_{S} \times N_{D}$ matrix indexed by $S \times D$, whose entries are $W(s, d)=\left(W_{1}(s, d), W_{2}(s, d)\right)$. $W$ is called the Wiring matrix. For convenience, we will refer to an SHI by its size $\left(c_{T}, c_{R} ; N_{S}, N_{D}\right)$ whenever there is no possibility of confusion.

Messages are transmitted at discrete time slots, and are all one slot in duration. The reception rules are those of a shared directional multichannel [6], namely: a message is received successfully by a DS iff that DS is the addressee and the receiver that hears the transmission hears no other transmissions in the same time slot (no collision).

A round-robin Transmission Schedule is an $N_{S} \times N_{D}$ matrix $X$ indexed by $S \times D$ whose entries $X(s, d)$ have time-slots as values. A schedule $X$ is compatible with a wiring $W$ if and only if for all $(s, d) \in S \times D, s$ communicates successfully with $d$ at time $X(s, d)$. Formally, a schedule $X$ is compatible with a wiring $W$ iff for any two pairs $\left(s_{1}, d_{1}\right) \neq$ $\left(s_{2}, d_{2}\right) \in S \times D$ the following holds: if both $W_{1}\left(s_{1}, d_{1}\right)=W_{1}\left(s_{1}, d_{2}\right)$ and $W_{2}\left(s_{1}, d_{2}\right)=$ $W_{2}\left(s_{2}, d_{2}\right)$, then $X\left(s_{1}, d_{1}\right) \neq X\left(s_{2}, d_{2}\right)$. Fig. 1 illustrates the compatibility rules.


Figure 1: Compatibility of wiring and schedule: since $W_{1}(1,1)=W_{1}(1,2)(=0)$ and $W_{2}(1,2)=W_{2}(2,2)(=1)$, transmissions from $\mathrm{SS}_{1}$ to $\mathrm{DS}_{1}$ and from $\mathrm{SS}_{2}$ to $\mathrm{DS}_{2}$ must not be scheduled for the same slot. (Collision at $\mathrm{DS}_{2}$.)

Finally, $f\left(c_{T}, c_{R} ; N_{S}, N_{D}\right)$ denotes the minimum number of different time slots for which there exist compatible wiring $W$ and transmission schedule $X$ (permitting one message from each SS to every DS $)$, and note that $f\left(c_{T}, c_{R} ; N_{S}, N_{D}\right)=f\left(c_{R}, c_{T} ; N_{D}, N_{S}\right)$ [6].
Example: An SHI with capacity $\log _{2} N[6]$
Consider $k$ SS's, each with two transmitters, and $2^{k}$ DS's, each with a single receiver. Let each SS be labeled with a $k$-bit vector containing a single ' 1 ', and each DS with a $k$-bit vector. Let $W(s, d)=(s \cdot d, 0)$. (Inner product of the two $k$-bit vectors, modulo 2 ; $W_{2}(s, d) \equiv 0$ because $c_{R}=1$.) Finally, let $X(s, d)=s+d$. (Bit-by-bit addition, modulo 2.) Thus, each SS partitions the DS's based on a different bit in their label. Fig. 2 depicts such an SHI for $k=4$. (Refer only to the top "layer" of SS's.)

This interconnection allows one message to be sent from each SS to each DS at a rate of $k$ per slot [6]. As depicted in Fig. 2, this interconnection can be augmented to form a ( 2,$1 ; 2^{k}, 2^{k}$ ) SHI. We note in passing that a modification that permits one additional concurrent transmission was presented in [8].


Figure 2: An interconnection of $2^{k}$ 2-transmitter SS's and $2^{k}$ single-receiver DS's. ( $k=4$.) The SS's are of $k$ "types", each wired differently, with $2^{k} / k$ identically-wired SS's in each type. Rectangles and circles denote stations and star couplers, respectively.

## 3. Random-access schemes

Random access schemes such as ALOHA and CSMA-CD are attractive, since they help allocate the transmission media in an adaptive way, and sharply reduce low-load delay. In this section, we address the question of whether such schemes can also attain the high degree of concurrency possible with round-robin schedules at heavy, uniform loads.

### 3.1 Candidate access schemes

CSMA and CSMA-CD cannot be used with non-bus-oriented SHI's, since collisions are only defined at individual receivers, which hear different subsets of transmitters.

With slotted ALOHA, the maximum uniform-traffic throughput attainable by any single-path SHI is $\frac{1}{e} \cdot c_{T} \cdot c_{R}$, so the advantage of non-bus-oriented SHI's is lost [3][6]. Slotted ALOHA can, nevertheless, be used at very low loads to obtain low delay or adapt to skews in the traffic pattern.

Busy-tone multiple-access, BTMA for short [16], was originally invented as a solution to the "hidden terminal" problem in packet radio: station A can hear B and C and they can hear it, but a mountain prevents B and C from hearing each other, so they may cause collisions at A. The proposed solution was to have the receiving station transmit a busy tone on a side channel, and a station hearing a busy tone would not transmit.

BTMA is directly applicable to non-bus-oriented SHI's, since "hidden terminal" situations are abundant by design. To implement this scheme, each receiver would be equipped with a small busy-tone transmitter that transmits along the same fiber in the reverse direction. Similarly, each transmitter would be equipped with a busy-tone sensor. The busy tone emitted by a receiver reaches exactly those transmitters which, by transmitting, could cause a destructive collision at that receiver. To avoid unnecessary blocking of transmitters, this should be done only by receivers that are receiving a packet addressed to them.

In the remainder of this section, we explore the merits of BTMA, focusing on heavy load and a uniform traffic pattern, to see whether BTMA should be used exclusively across the load range.

### 3.2 Uniform-traffic capacity with BTMA

We begin by deriving an upper bound, and then show that it is tight. The derivations will be made for a more general setting of a shared directional multichannel, and then converted to SHI's among stations with multiple transmitters and receivers.

A shared directional multichannel [6], SDM for short, comprises a set of inputs, to which we connect transmitters, a set of outputs to which we connect receivers, and a specification of connectivity between inputs and outputs. For our purposes, we consider SDM's with $t$ inputs and $r$ outputs, such that each input is connected to exactly $d_{T}$ outputs and each output is connected to exactly $d_{R}$ inputs. Clearly, $t \cdot d_{T} \equiv r \cdot d_{R}$.

To obtain an upper bound, we use an "idealized" BTMA: zero propagation delay and no race conditions. In each time slot, we order the transmitters that have a packet for transmission at random, and begin to execute BTMA in the chosen order. Each transmitter is assumed to pick a random destination, and the choices are assumed to be independent from transmitter to transmitter and from slot to slot. This scheme is collision-free in the sense that a transmission of an "early" transmitter cannot be interfered with by that of a later one. Consequently, its throughput is monotonically non-decreasing with offered load. Therefore, we will assume that all $t$ transmitters wish to transmit in every slot. Note that a transmitter may unknowingly transmit a message addressed to a blocked destination, since only receivers engaged in the reception of a message addressed to them emit a busy tone.

Since a transmitter does not base its decision to actually transmit on the destination of its message, the most optimistic assumption one can make is that every receiver hears exactly one transmission, and that this transmission is intended for it with probability $1 / d_{T}$. Expected throughput is thus bounded from above by $r / d_{T}$.

Casting this result in the context of the SHI's among stations, $r=N_{D} \cdot c_{R}$ and $d_{T}=N_{D} / c_{T}$. Consequently, $S_{B T M A} \leq c_{T} \cdot c_{R}$.

Proposition 1. The capacity of any SHI connecting source stations, each with $c_{T}$ transmitters, to destination stations, each with $c_{R}$ receivers, with busy-tone multiple-access (BTMA) is at most $c_{T} \cdot c_{R}$.

The tightness of this upper bound can easily be proved using bus-oriented SHI's, in which $c_{T} \cdot c_{R}$ independent subnetworks are constructed, each with a throughput of one for idealized BTMA. We conjecture that this is also the capacity of any non-bus-oriented single-path SHI with idealized BTMA.

Although the performance of BTMA under heavy load is clearly disappointing, it is useful at low loads and skewed traffic pattern. Unlike ALOHA, it is stable and throughput is monotonically non-decreasing with offered load, but one must decide whether the added complexity is warranted.

## 4. Schedule folding

### 4.1 Folded schedules

In the basic schedules that attain the high capacity, each schedule slot is allocated to a set of (SS,DS) pairs which may communicate concurrently in that slot, and are all guaranteed to succeed. The idea in a "folded" schedule is that each slot would be allocated to a set of equisized groups of (SS,DS) pairs such that if at most one (arbitrary) member of each group attempts to use the slot, all will succeed. High capacity is retained by selecting the groups such that there is no intergroup interference. Contention among group members for any given slot in which that group is allowed to communicate may be carried out in a variety of ways, including ALOHA. The heavy-load, uniform traffic throughput would thus be equal to that with the unfolded schedule, multiplied by the "utilization factor" of the access scheme used by group members to share a slot.
Definition. Shortest folded schedule. Let $f^{*}\left(c_{T}, c_{R} ; N_{S}, N_{D}\right)$ be the minimum number of different time slots for which there exist a compatible wiring $W$ and transmission schedule $X$ such that every ( $S S, D S$ ) pair is permitted to communicate in some slot (albeit with no guarantee of success), and the heavy-load, uniform-traffic capacity of the interconnection is equal to the maximum capacity with any unfolded schedule to within a constant factor.
Definition. Folding factor. We refer to $f() / f^{*}()$ as the folding factor, and note that it is equal to the aforementioned group size.

### 4.2 The folding factor of a given interconnection

Consider an "expanded" wiring matrix of size $\left(c_{T} N_{S} \times c_{R} N_{D}\right)$. Each of its entries is a single bit, denoting whether a given transmitter can be heard by a given receiver. Partition the matrix into groups of (not necessarily contiguous) identical rows. Based on symmetries in the design of the SHI's under consideration, we can assume that the groups of identical rows are all of equal sizes. Next, repeat the process for columns (the same symmetries apply).

Lemma 2. The maximum folding factor for a given interconnection is equal to the product of the sizes of the row-groups and column-groups in the wiring matrix.

Proof. Identical rows represent transmitters that are heard by identical sets of receivers. Thus, if in a given slot one of these transmitters is allowed to transmit to a given destination, letting a different one in the group transmit instead to the same destination would have the same effect (success, blocking, etc.). Similarly, identical columns represent receivers that hear identical sets of transmitters. Therefore, if a transmission intended for one of them is successful, it would be successful if it were intended for any one of the others. So, given a specific (transmitter, receiver) pair that are guaranteed to succeed in a given slot, the transmitter can be replaced with any other transmitter in its group, and the receiver can be replaced with any receiver in its group. The number of (source, destination) pairs that can communicate in this slot (though not concurrently) is therefore the product of the transmitter and receiver group sizes times the concurrency of the interconnection, and the folding factor is simply the product of the group sizes.

Example. Consider the $\left(2,1 ; k, 2^{k}\right)$ SHI described earlier. From its construction (see Fig. 2) it is clear that no two transmitters are heard by identical sets of receivers, and no two receivers hear identical sets of transmitters. Thus, the folding factor is one, and the minimum schedule length is $k \cdot 2^{k} / k=2^{k}$. In the $\left(2,1 ; 2^{k}, 2^{k}\right) \mathrm{SHI}$ constructed from it by adding "layers" of SS's, transmitter-group sizes are all $2^{k} / k$; the length of the folded schedule is $2^{k} \cdot 2^{k} /\left(k \cdot 2^{k} / k\right)=2^{k}$, which is equal to the number of stations. (Same as the length of a conventional TDMA round!)

### 4.3 Folding factor for composite SHI's

Given a wiring matrix, one can easily discover the maximum folding factor. Nevertheless, we proceed to derive expressions for it in order to gain more insight.

Large, high-concurrency SHI's can be constructed by combining smaller ones to form "product" interconnections [6]. We therefore next derive the folding factor of a product interconnection as a function of the folding factors of the constituent interconnections. (The folding factor for SHI's with a single transmitter per station or a single receiver per station is either one or can be trivially derived, as in the foregoing example.) We begin by briefly describing the method by which interconnections can be composed to form larger ones in a manner that retains the high concurrency properties [6].

Consider two interconnections, indexed by $i \in\{1,2\}$. Let $S_{i}, D_{i}, T_{i}, R_{i}$ satisfy $\left|S_{i}\right|=$ $N_{S_{i}},\left|D_{i}\right|=N_{D_{i}},\left|T_{i}\right|=c_{T_{i}}$, and $\left|R_{i}\right|=c_{R_{i}}$, where, as before, $T_{i}$ indexes the transmitters of each station in $S_{i}$ and $R_{i}$ indexes the receivers of each station in $D_{i}$. Let $W^{i}=\left(W_{1}^{i}, W_{2}^{i}\right)$ and $X^{i}$ be compatible wiring and scheduling matrices for the communication between $S_{i}$ and $D_{i}$, such that $W_{1}^{i}\left(s_{i}, d_{i}\right) \in T_{i}$ and $W_{2}^{i}\left(s_{i}, d_{i}\right) \in R_{i}$ for $\left(s_{i}, d_{i}\right) \in S_{i} \times D_{i}$.
Let $S=S_{1} \times S_{2}, D=D_{1} \times D_{2}, T=T_{1} \times T_{2}$ and $R=R_{1} \times R_{2}$. We construct wiring and scheduling matrices for communication between $S$ and $D$, where $T$ indexes the transmitters of each station in $S$, and $R$ indexes the receivers of each station in $D$, as follows. For a pair of stations $s=\left(s_{1}, s_{2}\right) \in S_{1} \times S_{2}, d=\left(d_{1}, d_{2}\right) \in D_{1} \times D_{2}$, we let $W(s, d)=\left(W_{1}(s, d), W_{2}(s, d)\right)=\left(\left(W_{1}^{1}\left(s_{1}, d_{1}\right) W_{1}^{2}\left(s_{2}, d_{2}\right)\right),\left(W_{2}^{1}\left(s_{1}, d_{1}\right) W_{2}^{2}\left(s_{2}, d_{2}\right)\right)\right)$ and $X(s, d)=\left(X^{1}\left(s_{1}, d_{1}\right) X^{2}\left(s_{2}, d_{2}\right)\right)$, where $X^{1} X^{2}$ denotes concatenation of the two strings to form a larger one or a higher-dimensional vector; similarly for $W_{i}^{1} W_{i}^{2}$.

Lemma 6 in [6] states that the foregoing schedule and wiring are compatible. Also, the length of the round for the composite interconnection is equal to the product of the lengths of the rounds of the constituent interconnections.

Theorem 3. The maximum folding factor for a composite interconnection is equal to the product of the maximum folding factors for the constituent interconnections.

## Proof.

Lower bound. In Lemma 6 of [6] there is no restriction on the constituent schedules, other than the need for them to be compatible with their own wiring matrices. It follows immediately that the lemma applies to folded schedules as well. Thus, the length of the composite folded schedule will not exceed the product of the lengths of the individual folded schedules, each of which is in turn equal to the length of the respective unfolded schedule divided by the respective folding factor. So, the maximum composite folding factor is greater than or equal to the product of the individual ones. Upper bound. In [15], it has been shown that one possible layout of a composite interconnection comprises a "column" containing an appropriate number of one of the
constituent interconnections, followed by a column containing an appropriate number of the other. For simplicity of exposition, we refer to the interconnections in the two columns as \#1 and \#2 interconnections, respectively. Each input of a \#1 interconnection is connected to a single transmitter, each output of a $\# 2$ interconnection is connected to a single receiver, and there is exactly one connection between each \#1 interconnection and every \#2 interconnection. Moreover, each output of a \#1 interconnection is connected to a single input of a \#2 interconnection, and each input of a \#2 interconnection is connected to a single output of a \#1 interconnection. (The restrictions are mandated by single-hop connectivity of each of the constituent SHI's as well as the composite one, and the provision of a single path between any two stations.) Since all layouts must be equivalent in terms of end-to-end connectivity and disjointness of paths, anything deduced from this layout is valid for all layouts of a composite interconnection.
Since actual transmitters (receivers) are only connected to \#1 (\#2) interconnections, we will use input (output) groups to refer to ports of the constituent interconnections to which, in stand-alone operation, one would connect the transmitters (receivers) belonging to the same transmitter (receiver) group.
Let $\left|I G_{1}\right|$ and $\left|I G_{2}\right|$ denote the sizes of input groups of a \#1 and \#2 interconnection, respectively. For transmitters that can be heard by a given receiver to be in the same transmitter group (of the composite interconnection), they must all reach that receiver through the \#2 interconnection to which it is connected. Moreover, they must all do so via the same input group of this $\# 2$ interconnection; i.e., through at most $\left|I G_{2}\right|$ of its inputs. Also, since there is exactly one connection between any \#1 and \#2 interconnection, transmitters in the same transmitter group (of the composite interconnection) which are connected to the same $\# 1$ interconnection must all be connected to a single input group of that interconnection (otherwise they will reach different \#2 interconnections and consequently different receivers).
So, each of the $\left|I G_{2}\right|$ inputs of the $\# 2$ interconnection can be reached by at most $\left|I G_{1}\right|$ transmitters in a common transmitter group. Consequently, the total number of transmitters in a transmitter group of the composite interconnection is at most $\left|I G_{1}\right| \cdot\left|I G_{2}\right|$. A similar argument applies to receivers. Combined with Lemma 2, it follows that the folding factor cannot exceed the product of those of the constituent interconnections.

### 4.4 Concurrency - folding-factor trade-off

In this section, we use a simple example to show a non-trivial tradeoff between capacity and folding factor in composing interconnections. Let us use the following basic building blocks:

- $\left(2,1 ; k, 2^{k}\right)$ : concurrency $k$; no folding possible; $f=2^{k}$.
- single bus with an arbitrary number of stations: concurrency one, $f^{*}=1$.

Consider the following two compositions, both producing ( 2,$2 ; N, N$ ) SHI's:
Composition 1. $\left(2,1 ; k, 2^{k}\right) X\left(1,2 ; 2^{k}, k\right) \rightarrow\left(2,2 ; k 2^{k}, k 2^{k}\right)$.

- $f()=2^{2 k} ; \quad f^{*}()=2^{2 k}>N^{2} /\left(\log _{2} N\right)^{2} \quad($ close for large $N)$
- concurrency: $k^{2}$ (approximately $\left.\log _{2}^{2} N\right) ; \quad f() / f^{*}()=1$.

Composition 2. This is a two-step construction:
Step 1. $\left(2,1 ; k, 2^{k}\right) X\left(1,1 ; 2^{k} / k, 1\right) \rightarrow\left(2,1 ; 2^{k}, 2^{k}\right)$.
$f()=2^{k} \cdot 2^{k} / k=2^{2 k} / k ; f^{*}()=2^{k} \cdot 1=2^{k}$. Concurrency: $k$.
Step 2. $\left(2,1 ; 2^{k}, 2^{k}\right) X\left(1,2 ; 2^{k}, 2^{k}\right) \rightarrow\left(2,2 ; 2^{2 k}, 2^{2 k}\right)$.

- $f()=2^{4 k} / k^{2} ; \quad f^{*}()=2^{2 k} \quad(=N)$
- concurrency: $k^{2}\left(=\log _{2}^{2} N / 4\right) ; \quad f() / f^{*}()=2^{2 k} / k^{2}=4 N / \log _{2}^{2} N$.

For equal values of $N$, the second composition thus yields a much shorter folded schedule, at a penalty of at most a factor of four in concurrency. From this example, Lemma 6 in [6] and Theorem 3, it thus follows that high-capacity SHI's can be composed whose folded schedule lengths are equal to the number of stations.

## 5. Conclusions

This paper explored schemes for operating high-concurrency, non-bus-oriented SHI's: BTMA and "folded" round-robin schedules.
An idealized form of BTMA gives all receivers an equal chance in every slot, but a small one, to successfully receive a message, and fails to take advantage of the potential of non-bus-oriented SHI's for high concurrency. Deterministic scheduling, in contrast, causes collisions at most receivers but is able to guarantee success to many more than would succeed on average with BTMA.

Schedule "folding" and hierarchical operation (concurrent, non-interfering permission rights to groups of (source, destination) pairs, with contention within each group) were shown to dramatically reduce round length and thus low-load delay, while retaining the high concurrency. This also reduces sensitivity to skews in the traffic pattern.

Further issues to be explored include the impact of folding on delay at various loads and on the sensitivity of throughput to traffic patterns, and dynamic scheduling (picking the schedule slot to be executed in the next time slot), particularly with non-uniform traffic patterns. Finally, an investigation of an unrealistic BTMA, in which receivers emit a different busy tone when blocked though not receiving and a transmitter can tell which receivers are emitting such a tone, may further improve our understanding of what is necessary for attaining a high degree of concurrency.

## References

[1] M.A. Marsan and D. Roffinella, "Multichannel local area network protocols", IEEE JSAC, vol. 1, no. 5, pp. 885-897, Nov. 1983.
[2] T. Lang, M. Valero and M.A. Fiol, "Reduction of Connections for Multibus Organization," IEEE Trans. Comp., vol. C-32, no. 8, pp. 707-715, Aug. 1983.
[3] Y. Birk, Concurrent Communication among Multi-Transceiver Stations via Shared Media, Ph.D Dissertation, Electrical Engr. Dept., Stanford Univ., Dec. 1986. Also available as tech. rep. CSL-TR-87-321, Mar. 1987.
[4] Y. Birk, F.A. Tobagi and M.E. Marhic, "Bus-oriented interconnection topologies for single-hop communication among multi-transceiver stations", Proc. IEEE INFOCOM '88, March 1988.
[5] T.E. Stern, "Linear lightwave network," CTR Tech. Rep. No. 184-90-14, Columbia Univ., 1990.
[6] Y. Birk, N. Linial and R. Meshulam, "On the Uniform-Traffic Capacity of SingleHop Interconnections Employing Shared Directional Multichannels", IEEE Trans. IT, vol. 39, no. 1, pp.186-191, Jan. 1993. (Also IBM Res. Rep. RJ 7859 (72519)).
[7] J. Kilian, S. Kipnis and C.E. Leiserson, "The Organization of Permutation Architectures with Bussed Interconnections", Proc. 28th Annual Symp. on Foundations of Comp. Sci., Los Angeles, CA, Oct. 1987.
[8] M.T. Busche, B. Hajek, "On optical interconnection of stations having multiple transmitters and receivers", Proc. 1990 Int. Symp. on Information Theo. and its Applications (ISITA '90), Hawaii, pp. 967-970, Nov. 1990.
[9] R. A. Barry, "Wavelength routing for all-optical networks", Ph.D Dissertation, EECS Dept., Mass. Inst. of Tech., Sep. 1993, pp. 153-160.
[10] G. Pieris and G. Sasaki, "Scheduling transmissions in WDM broadcast and select networks," Proc. 31st Allerton Conference, Sep. 1993 (to appear).
[11] C. Skawratananond, "Design methodology for bus-oriented networks with single-hop constraint," M.S. Thesis Dept. of Electrical Engr., U. of Texas at Austin, Aug. 1992.
[12] Y. Birk, "Fiber-Optic Bus-Oriented Single-Hop Interconnections among MultiTransceiver Stations," IEEE JLT, vol. 9, no. 12, pp.1657-1664, Dec. 1991.
[13] Y. Birk, "Efficient layout of a passive, single-hop fiber-optic interconnection (among $N$ stations) with capacity $\log _{2} N^{\prime \prime}$, in Proc. 29th Annual Allerton Conf. on Commun., Control and Computing, Oct. 1991, Monticello, IL, pp. 322-331.
[14] Y. Birk, "Power-Efficient Layout of a fiber-Optic Multistar that Permits $\log _{2} N$ Concurrent Baseband Transmissions among N Stations," IEEE JLT, Special issue on broadband optical networks, 1993 (to appear).
[15] Y. Birk, "Power-optimal layout of passive, single-hop, fiber-optic interconnections whose capacity increases with the number of stations", Proc. IEEE INFOCOM '93.
[16] F.A Tobagi and L. Kleinrock, "Packet Switching in Radio Channels: Part II - The Hidden Terminal Problem in Carrier Sense Multiple-Access and the Busy Tone Solution." IEEE Trans. Commun. vol. 23, no. 12, pp.1417-1433, Dec. 1975.


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