"SUPERNODES" IN PACKET RADIO NETWORKS EMPLOYING CODE DIVISION MULTIPLE ACCESS

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ABSTRACT

CDMA provides new ways of allocating network resources to network nodes so as to match nonuniform traffic requirements. The enhancement of a node's throughput as a result of equipping it with several receivers, several transmitters or both is studied for the *slotted ALOHA* access scheme. It is also shown that funneling all of a node's inbound traffic via two of its neighbors can increase that node's inbound throughput by up to 36% without any additional hardware and with simple and robust protocol support.

I. INTRODUCTION

In a real network, certain nodes, such as gateways, command and control posts, etc., must carry much more traffic than most other nodes. It is clearly desirable to allocate more resources to such nodes than to others, thereby turning them into "**supernodes**".

In narrowband (non *CDMA*) networks, the allocation of transmission rights can be used to give a node anything from no usage to exclusive usage of the channel. This is not true for *Code Division Multiple Access (CDMA)*, since a single transmission no longer uses up the entire capacity. The use of *CDMA*, on the other hand, allows for the concurrent reception (at the same location) of several packets, which are separable due to the use of different codes or to *time capture*. This introduces the option of equipping supernodes with **multiple transmitters, multiple receivers**, or both. *Power capture*, which is also possible in narrowband networks, is different since it arbitrates rather than allowing concurrency.

Viewing a network as a graph whose nodes correspond to network nodes, there is a link from node i to node j with tag k if and only if node j can hear transmissions of node i and has a receiver for code k. In networks employing *CDMA with Receiver-Directed Codes (CDMA/RDC)*, whereby nodes are allocated disjoint sets of receiver codes, we observe that each transmission activates only one link of the network graph. It is therefore possible to **mask individual links** of the graph. This is different from narrowband networks, in which the decision whether or not to use links is made jointly for all the outgoing links of a node.

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In the sequel, we consider a *CDMA/RDC* network and demonstrate our ideas for the case of a single supernode S in an otherwise densely populated but lightly loaded region of a packet radio network. Our goal is to increase the throughput of this node. In section II, we introduce the network model. In sections III and IV, we study the allocation of *multiple receivers* and *multiple transmitters* to the supernode in order to increase its inbound and outbound throughput, respectively. Gitman [4] has looked into multiple transmitters in a narrowband network, using directional antennas; however, his results do not apply to *CDMA*. In section V we combine multiple receivers and multiple transmitters. Section VI is devoted to the use of link masking for increasing the supernode's throughput. Section VII summarizes the work.

II. NETWORK MODEL

In order to permit a quantitative evaluation of various design options, while keeping the analysis simple, we use the *Slotted ALOHA* multi-access scheme, with packet lengths of exactly one slot. Time capture and power capture are not accommodated.

The supernode S has T transmitters and M receivers, with a different code for each receiver. S's region is modeled as follows:

- The region consists of S, N neighbors of S and other, *external*, nodes. Each of S's neighbors can hear transmissions of Q other neighbors and its transmissions can be heard by Q other neighbors.
- The transmission process of each of the N neighbors is Bernoulli (p) and is independent from node to node. A neighbor's transmission uses a supern-

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ode code with probability α, the code of any given neighbor (among the Q that can hear the transmission) with probability $\beta \cdot \frac{1-\alpha}{Q}$, and the code of an external node with probability $(1-\beta)(1-\alpha)$.

- In each time slot, with probability $P_{ET}(k)$, any given neighbor hears k ***external transmissions***; i.e., transmissions from external nodes. Such a transmission uses that neighbor's code with probability δ.
- $\mathcal{S}$ transmits in any given slot with probability p_0; when it transmits, it does so with all T transmitters, each using a different code.

The following conditions must be satisfied for a node to receive a packet with code k in a given slot: (*i*) the node is not transmitting, (*ii*) it does not hear any other packets with the same code (no ***intracode interference***), and (*iii*) the total number (l) of packets heard by the node, regardless of code, is not "excessive"; (we use $P_S(l)$ to express the probability that l is not "excessive", and refer to it as the channel capacity function. The capacity of the channel can be defined as the largerst number L for which $P_S(L)$ is greater than some constant. The phenomenon of a packet being destroyed due to finite channel capacity will be referred to as ***intercode interference***).

III. MULTIPLE RECEIVERS

The average throughput into the supernode is given by

$$S_{in} = (1-p_0)\sum_{l=1}^{N} \alpha l P_S(l)\binom{N}{l}p^l(1-p)^{N-l}\left(1-\frac{\alpha}{M}\right)^{l-1} \qquad (1)$$

For the case that all transmissions of the neighboring nodes are intended for $\mathcal{S}$, and $P_S(l) = 1$ for $0 \le l \le L$ and 0 otherwise, the dependence of p_{opt} (the value of p that maximizes S_{in}) on M and L is shown in Fig. 1, and Fig. 2 shows $S_{in}/(1-p_0)$ versus M with $p = p_{opt}(N, M, L)$. The first, steep portion of each curve represents the domain in which ***intracode*** interference is the dominant factor; the flat portion represents the situation in which ***intercode*** interference becomes the dominant factor. We observe that, while inbound throughput is initially proportional to the number of supernode receivers, the marginal benefit eventually tapers off due to ***intercode*** interference.

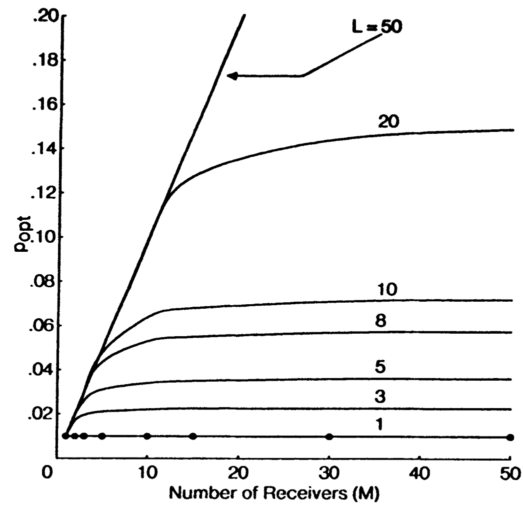

Fig. 1. Multiple receivers: neighbor's optimal probability of transmission. $N = 100$

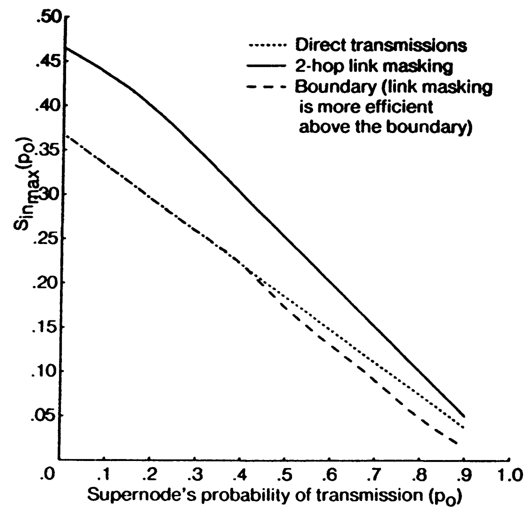

Fig. 2. Maximal normalized inbound throughput with multiple receivers. $N = 100$; $p = p_{opt}(N, M, L)$

IV. MULTIPLE TRANSMITTERS

We enforce two forms of synchronization between the T supernode transmitters: ***code synchronization***, whereby concurrent transmissions by the supernode's transmitters employ different codes, and ***time synchronization***, whereby the T transmitters are either all idle or all transmitting. Without time synchronization, $\mathcal{S}$ would hardly ever be available for reception.

Given the number of transmissions by each type of nodes ($\mathcal{S}$; neighbors; external nodes), ***intercode*** and ***intracode*** interference are independent of each other.

Consequently, given that the intended neighbor hears exactly $q \leq Q$ transmissions of other supernode neighbors along with T supernode transmissions and k external transmissions:

$$P[\text{ reception of a given supernode packet}|q,k,T] = (1-p)\left[\left[1-\beta\frac{1-\alpha}{N-1}\right]^q (1-\delta)^k\right] P_S(q+k+T) \tag{2}$$

Relaxing all the conditions except for the T supernode transmissions, we obtain:

$$P[\text{reception of a given supernode packet }|T] = (1-p)\cdot\sum_{q=0}^{Q}\sum_{k=0}^{\infty}\left\{\left[\left(1-\beta\cdot\frac{1-\alpha}{Q}\right)^q\cdot(1-\delta)^k\right] P_S(q+k+T)\cdot\left[\binom{Q}{q}\cdot p^q\cdot(1-p)^{Q-q}\cdot P_{ET}(k)\right]\right\} \tag{3}$$

which, when multiplied by $(T\cdot p_0)$, yields $\mathcal{S}$'s outbound throughput S_{out}.

Fig. 3 presents the results for a fully connected network with $\alpha = 1$ and $P_S(l) = 1$ for $0 \leq l \leq L$ and 0 otherwise. The dashed curves show the dependence of T_{opt} (the value of T that maximizes S_{out}) on p and L; T_{opt} is independent of p_0. The solid curves show the dependence of S_{out}/p_0 on p and L with $T = T_{opt}$. Note that the slopes of the solid curves are steeper than those of the dashed curves, indicating that the throughput per transmitter also decreases with an increase in p, even if the optimal number of transmitters is used. While the results would vary with N, the primary dependence is on $p \cdot N$, which indicates the fraction of the channel capacity that is unavailable for $\mathcal{S}$'s transmissions. It is also interesting to observe that the throughput per transmitter increases as we decrease T (and keep everything else unchanged). Consequently:

$$S_{out}(T) \geq \frac{T}{T_{opt}}\cdot S_{out}(T_{opt}), \quad T < T_{opt} \tag{4}$$

The oppsite is true for $T > T_{opt}$.

V. COMBINING MULTIPLE RECEIVERS AND MULTIPLE TRANSMITTERS

In sections III and IV, our goal was to maximize throughput in one direction, assuming that the parameters associated with the other direction had been set and are thus part of the environment. We now combine the two directions and address the problem

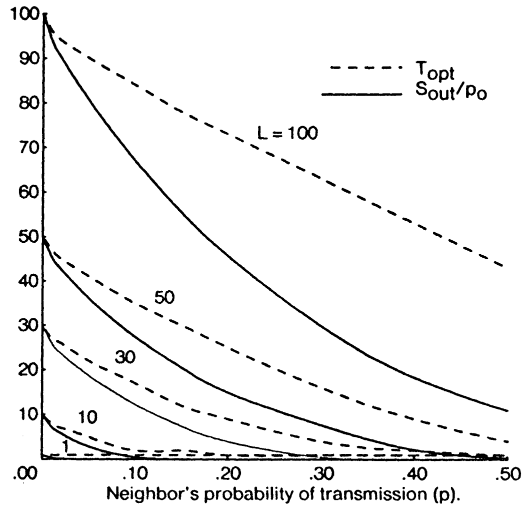

Fig. 3. Optimal number of supernode transmitters and maximal normalized outbound throughput. $N = 100$

of maximizing S_{in} and S_{out} subject to the constraint $S_{in}/S_{out} = \gamma$. In its most general form, this is a multidimensional optimization over the parameters M, T, p and p_0, (N and L are given). We will assume that M is also given, since not all the factors that limit M have been included in our model (e.g. code availability). One could also formulate several related problems. For example, there may be a cost constraint that determines $(M+T)$, and the goal will be to find $(M,\ T,\ p,\ p_0)$ that maximize throughput.

Since $S_{in_{max}}(p_0)$ decreases as p_0 increases, and since setting $T = T_{opt}(p)$ minimizes the value of p_0 required to achieve any given value of S_{out}, yet does not affect $S_{in}/(1-p_0)$ (because once $\mathcal{S}$ is transmitting it cannot receive, regardless of T), it follows that T should best be set to $T_{opt}(p)$, as computed earlier. Our problem thus reduces to a maximization of S_{in} over p, such that $S_{in}/S_{out} = \gamma$. In Fig. 4, we show $S_{in}/(1-p_0), S_{out}/p_0, p_0$, and $(S_{in}+S_{out})_{max}$ versus p for $\gamma = 1$. In Fig. 5 we show $p_{opt}, p_{0_{opt}}$ and $(S_{in} + S_{out})_{max}$ versus γ. Both figures were generated for the specific case that was used in sections III and IV.

The design problem becomes much more complicated when there are several supernodes, since transmissions intended for one supernode can be interfered with by transmissions intended for other supernodes as well as by transmissions of other supernodes. Consequently, S_{in} does depend on the number of transmitters used by the supernodes, and the optimal number of supernode transmitters is no longer the one

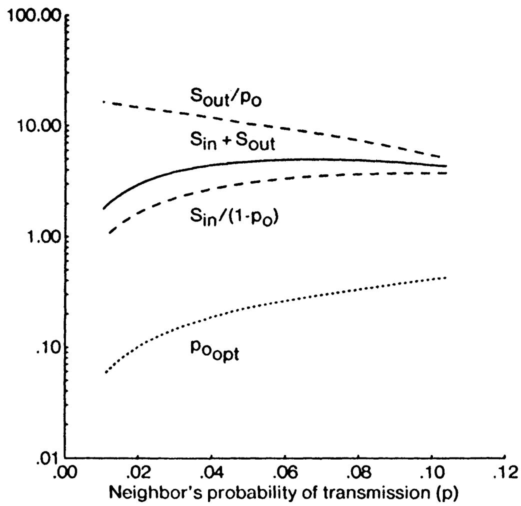

Fig. 4. Multiple receivers and transmitters: $S_{in} = S_{out}$; $N = 100$; $L = 20$; $M = 10$; $0 < p < p_{opt}(N, M, L, \text{inbound only})$.

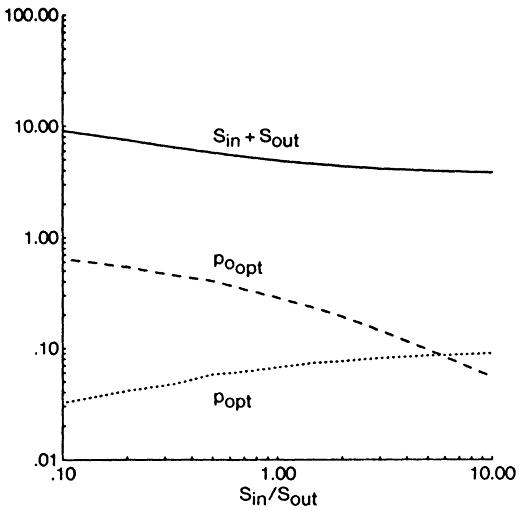

Fig. 5. Multiple receivers and transmitters: maximum throughput versus γ. $N = 100$; $L = 20$; $M = 10$.

obtained in section IV. In fact, it is smaller. While the optimization is multidimensional and, as a result, more complicated, the computation for each choice of parameters is similar to the simple case.

VI. LINK MASKING

Consider funneling all the inbound traffic destined for any given receiver of $\mathcal{S}$ through a subset of $\mathcal{S}$'s neighbors (***authorized neighbors*** for that receiver), thus masking $\mathcal{S}$'s remaining inbound links. Recalling that the throughput of a conventional *Slotted ALOHA* channel is $1/e$ for $N = \infty$ and 0.5 for $N = 2$, we can expect link masking to increase inbound throughput by up to 36%. For simplicity, we assume that $M = 1$ and $L = \infty$. The accommodation of multiple receivers is straightforward, provided that $N \geq 2M$; otherwise, it is slightly more complicated due to an overlap of the funnels for different receivers.

The analysis of link masking as applied to this specific example is similar to that of routing packets to a central node in a narrowband network via a sequence of repeaters [4,5,1]. In the referenced studies, it was assumed that the central node never transmits. We will study this for *CDMA/RDC*, allowing $p_0 \geq 0$.

Let us define the ***routing graph*** to be the directed graph consisting of the union of the paths to be used to route packets from each node to $\mathcal{S}$, but excluding the initial hop of those paths. Our goal is to determine the maximum attainable throughput into $\mathcal{S}$ and the simplest routing graph that can achieve it (shortest paths in terms of hops). Since throughput with Slotted ALOHA increases as the size of the contending population decreases, each node in the simplest routing graph should transmit to exactly one other node. Combining this with the requirement that all paths of the routing graph end at $\mathcal{S}$, we conclude that the simplest routing graph is a tree which has $\mathcal{S}$ as its root. Before proceeding, we must introduce some notation:

- *level* – The distance in hops from a node to $\mathcal{S}$ via the routing tree.
- n_i – The number of level-$(i+1)$ nodes that may transmit to each level-i node.
- p_i – Probability of transmission of any given level-i node in any given slot.
- P_{R_i} – Probability that any given level-i node receives a packet in any given slot.

A typical routing tree is shown in Fig. 6. The routing tree is governed by the following set of equations:

$$P_{R_i} = n_i p_{i+1}(1 - p_{i+1})^{n_i - 1}(1 - p_i) \qquad i = 0, 1, \ldots \tag{5}$$

$$P_{R_{i+1}} = \frac{P_{R_i}}{n_i} \tag{6}$$

Note that, in a narrowband network, assuming a symmetric hearing matrix that represents a tree, the right hand side of (5) would contain an additional factor of $(1 - p_{i-1})$, representing transmissions by the father of the level-i node [5].

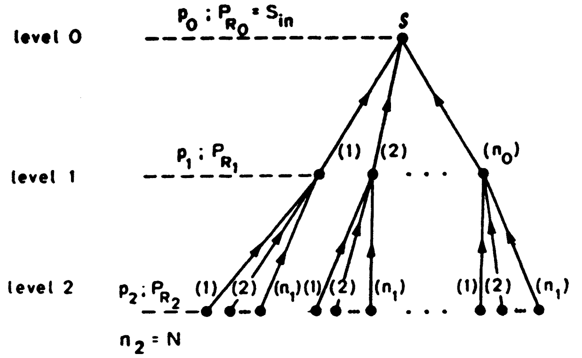

Fig. 6. Link masking: typical routing tree.

Assuming that the upper bound of $0.5(1-p_0)$ on $\mathcal{S}$'s inbound throughput is achievable, we proceed to impose requirements on the routing tree, beginning with level 0, and obtain upper bounds on n_i along with matching values of p_i. Obtaining $n_i \leq \infty$ is the indicator for having reached the leaves of the minimal routing tree.

level 0: we obtain: $n_0 = 2$; $p_1 = 0.5$;

level 1: combining this with (5) and (6), and noting that, for any value of n_i, P_{R_i} is maximized by setting $p_{i+1} = \frac{1}{n_i}$, yields:

$$\left(1 - \frac{1}{n_1}\right)^{n_1 - 1} = 0.5 \cdot (1 - p_0) \qquad (7)$$

Solving (7) for $\lfloor n_1 \rfloor$, we obtain the upper bound on n_1 as a function of p_0. We see that for $p_0 = 0$, $n_1 = 2$; for $p_0 \geq 0.264$ $(= 1 - 2/e)$, $n_1 \leq \infty$.

level 2: similarly, and recalling that $p_i \leq 0.5$, the upper bound on n_2 is ∞, regardless of p_0.

The two extremes of the required routing tree are shown in Fig. 7 . The size of the routing tree is independent of the number of network nodes. In the remainder of this section, we focus on link masking using a height-1 binary tree ($n_0 = 2$, $n_1 \leq \infty$, 2-hop paths); the 1st hop is from the source to an authorized neighbor, and the 2nd hop is from an authorized neighbor to $\mathcal{S}$.

When $p_0 < 0.264$, the 1st hop cannot support the maximal throughput of the 2nd hop (with $p_1 = 0.5$). For this case, $S_{in_{max}}(p_0)$ is calculated by substituting $n_0 = 2$ in (5) and in (6), replacing P_{R_1} in (6) with a lower bound of $\frac{1}{e}(1-p_1)$, and solving (5) and (6) for S_{in}. The lower bound is tight (it is exact for $n_1 = N = \infty$); the maximization is over p_1. When $0.264 \leq p_0 < 1$, the bottleneck is in the 2nd hop and $S_{in_{max}} = 0.5(1-p_0)$. We obtain:

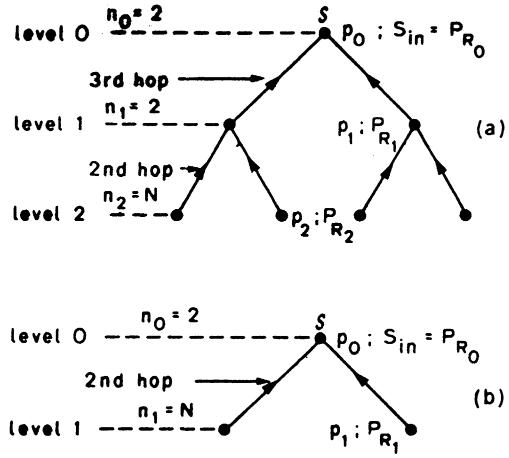

Fig. 7. Link masking: minimal routing trees. (*a*) $p_0 < (1-2/e)$; (*b*) $p_0 \geq (1-2/e)$.

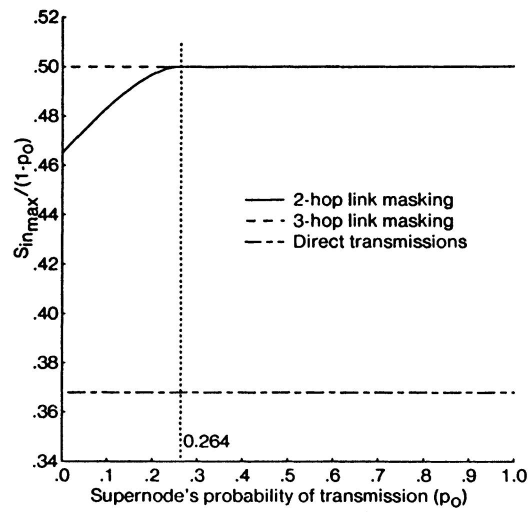

Fig. 8. Link masking: maximal normalized inbound throughput. $M = 1$; $N, L >> M$.

$$S_{in_{max}}(p_0) = \begin{cases} \frac{2}{e} \cdot \left(1 - \frac{e^{-1}}{1-p_0}\right), & 0 \leq p_0 \leq 0.264 \\ 0.5(1-p_0), & 0.264 < p_0 < 1 \end{cases} \qquad (8)$$

A plot of $S_{in_{max}}/(1-p_0)$ versus p_0 is shown in Fig. 8; results for direct transmissions and for 3-hop link masking are presented for reference. For any feasible pair of (p_0, S_{in}), p_1 is calculated from (5). Then, p_2 is calculated from (5),(6). Finally, p_2 must be doubled, since a level-2 node transmits to two level-1 nodes.

Since the capacity of real channels is finite, an

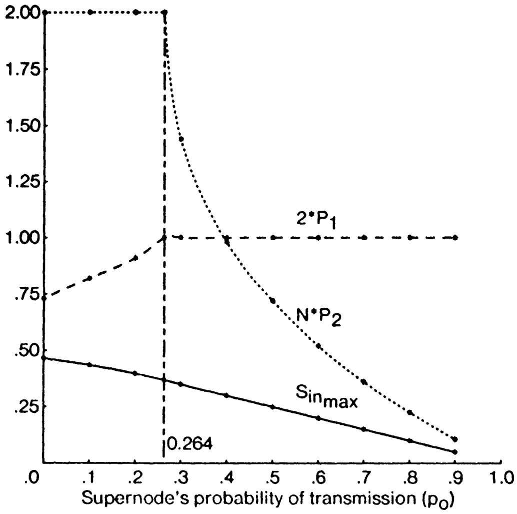

Fig. 9. Maximal inbound throughput and required transmission rates with 2-hop link masking. $M = 1$; $N, L >> M$.

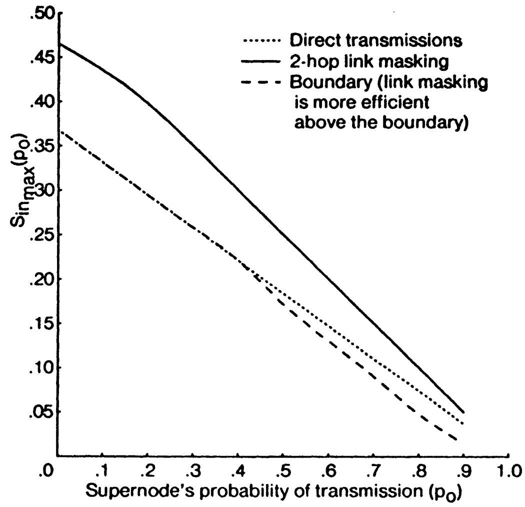

Fig. 10. Feasibility and superiority boundaries for direct transmissions and for 2-hop link masking. $M = 1$; $N, L >> M$.

important aspect of employing various schemes in the *CDMA* environment is the transmission rate that is required for achieving a given throughput. We proceed to compare 2-hop link masking to direct transmissions (single-hop) for this measure:

Direct transmission: let us set $p = \frac{\theta}{N}$. Assuming large N, we obtain:

$$\theta \cdot e^{-\theta} = \frac{S_{in}}{(1-p_0)} \qquad 0 \le S_{in} \le \frac{1}{e} \cdot (1-p_0);\ 0 < \theta \le 1 \tag{9}$$

and the transmission rate is θ.

2-hop link masking: the transmission rates on the 1st and 2nd hops are Np_2 and $2p_1$, respectively. In Fig. 9, we plot $S_{in_{max}}$ along with the aggregate transmission rate on each of the hops, as a function of p_0.

In Fig. 10, we plot $S_{in_{max}}(p_0)$ as a function of p_0, for direct transmissions and for 2-hop link masking. The feasible (p_0, S_{in}) combinations for each scheme are represented by the region under the appropriate curve. The dotted curve in that figure is the equal efficiency line. Below it, direct transmissions are more efficient (less transmissions per reception) than 2-hop link masking. Above the boundary, 2-hop link masking is more efficient. We observe that, for low p_0, direct transmissions are more efficient as long as they are feasible, and the boundary corresponds to $S_{in_{max}}(p_0)$ for direct transmissions. However, as p_0 increases, the boundary moves deeper into the feasible domain of direct transmissions. To understand this, note that increasing p_0 while keeping p (direct transmissions) and p_1 (2-hop link masking) unchanged has the same effect on the throughput of the two schemes, namely a decrease. The efficiency of the 2nd hop (link masking) relative to that of direct transmissions is also unchanged. However, due to the drop in S_{in}, p_2 (link masking) can be reduced, resulting in increased efficiency of the 1st hop. Consequently, there is an overall improvement in the efficiency of link masking relative to that of direct transmissions.

While intercode interference has been ignored in this section, its effect on the performance of link masking relative to that of direct transmissions is minute, since the aggregate transmission rate associated with a receiver employing link masking is at most 3, which is typically much smaller than channel capacity.

The results depend on N only when it becomes small. The indicator for the closeness of the approximation in assuming "very large N" is the relative difference between $(1-1/N)^{N-1}$ and $1/e$. For example, the differences for $N = 5, 10, 20$ are 11%, 5.3% and 2.5%, respectively.

Can link masking be employed efficiently by many busy nodes? One fundamental limitation stems from the fact that, for each receiver that masks some of its incoming links, there must be two other nodes that are not required to handle a significant volume of other traffic. Consequently, at most 1/3 of the nodes may apply link masking. In practice, the more severe limitation stems from the finite capacity of the channel.

Should link masking be used for outbound traffic? Assuming that the destinations of the outbound packets are not themselves busy nodes and that each packet has a single destination, the answer is a definite "no", because the probability of reception of a supernode packet by its lightly-loaded destination node is higher than the probability of reception by the busy forwarding node. Link masking may nevertheless be beneficial for multi-destination packets.

The protocol required to support 2-hop link masking is very simple and robust: since each of the N neighbors of $\mathcal{S}$ may transmit to either of the two authorized nodes, each network node keeps two addresses for $\mathcal{S}$, which are actually the addresses of the two authorized neighbors, and uses either one (at random). This has the additional benefit of balancing the load of the routing tree.

VII. CONCLUSION

We have shown that equipping a node with several receivers and transmitters achieves an increase in its inbound and outbound throughput, respectively, which is initially linear in the number of receivers transmitters, but is eventually limited by channel capacity.

Masking all but two of a node's incoming links has been shown to increase its inbound throughput by up to 36%. Even in the worst case ($p_0 = 0$), it came close to this upper bound. Consequently, we need never consider any more complicated routing graphs. Outbound throughput is also increased as a side effect. Unlike multiple receivers and multiple transmitters, link masking requires no additional hardware, since the funnels are constructed using (lightly utilized) existing hardware in neighboring nodes. Furthermore, at high throughput levels, particularly when the supernode itself transmits frequently, link masking uses up less of the channel capacity than do direct transmissions to the supernode, for the same inbound throughput. The protocol required to support link masking is very simple and robust.

The number of supernodes that can coexist (efficiently) in the same region of a network is limited primarily by channel capacity; the availability of codes could also be a limiting factor, but more often than not this is not the case [3].

In unslotted systems, time capture creates the option of allocating the same code to several receivers. In a forthcoming paper [2], we explore architectures and code-assignment policies for multi-receiver supernodes in an unslotted system with time capture. Since time capture only reduces intracode interference, the results of the sections dealing with multiple transmitters and receivers, pertaining to intercode interference, are also valid in unslotted systems. As for link masking, we note that time capture is a collision resolution mechanism of sorts. As such, it competes with link masking in the sense that the throughput into a single receiver is bounded by 1.0; if either mechanism achieves a significant improvement, there is not much left for the other. However, the contribution of link masking may still be significant in the case of very short packets with relatively long vulnerable periods (acquisition times).

We have obviously not exhausted the design flexibility offered by *CDMA*. For example, one could construct a ***super-link*** between two busy nodes that communicate extensively with each other. Also, most measures that can be taken to increase throughput in a narrowband network, such as spatial reuse of the channel or reservation schemes, are equally applicable to *CDMA*. Finally, our focus on *CDMA* should not be interpreted as a claim that *CDMA* is superior to other approaches. Rather, given that it is being used, we are attempting to get the most out of it.

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